



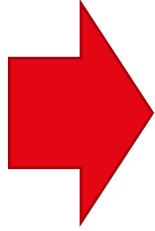
CIVIL-309: URBAN THERMODYNAMICS

**Assist. Prof.
Dolaana Khovalyg**

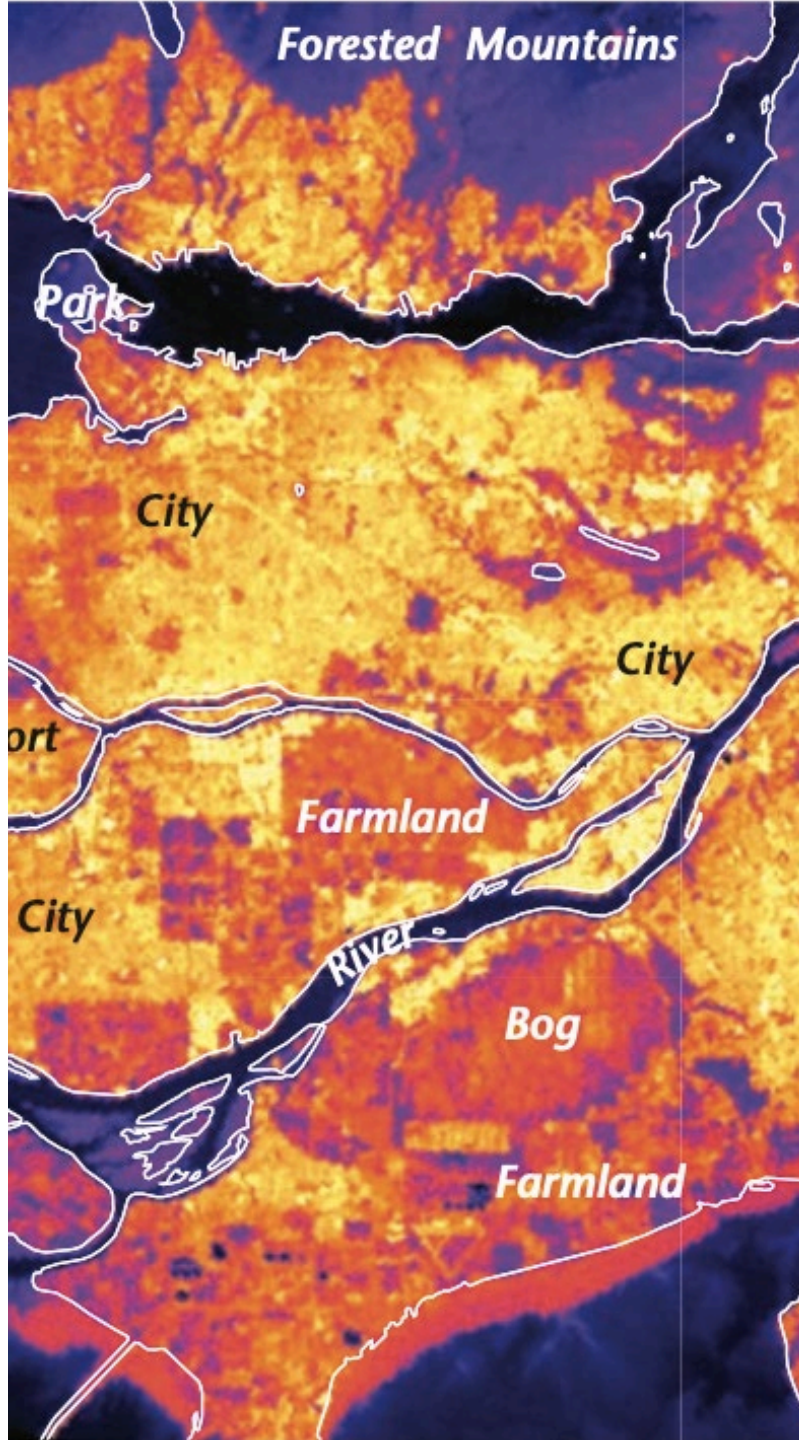
Lecture 08:

Water bodies-environment interaction

EPFL Course Schedule



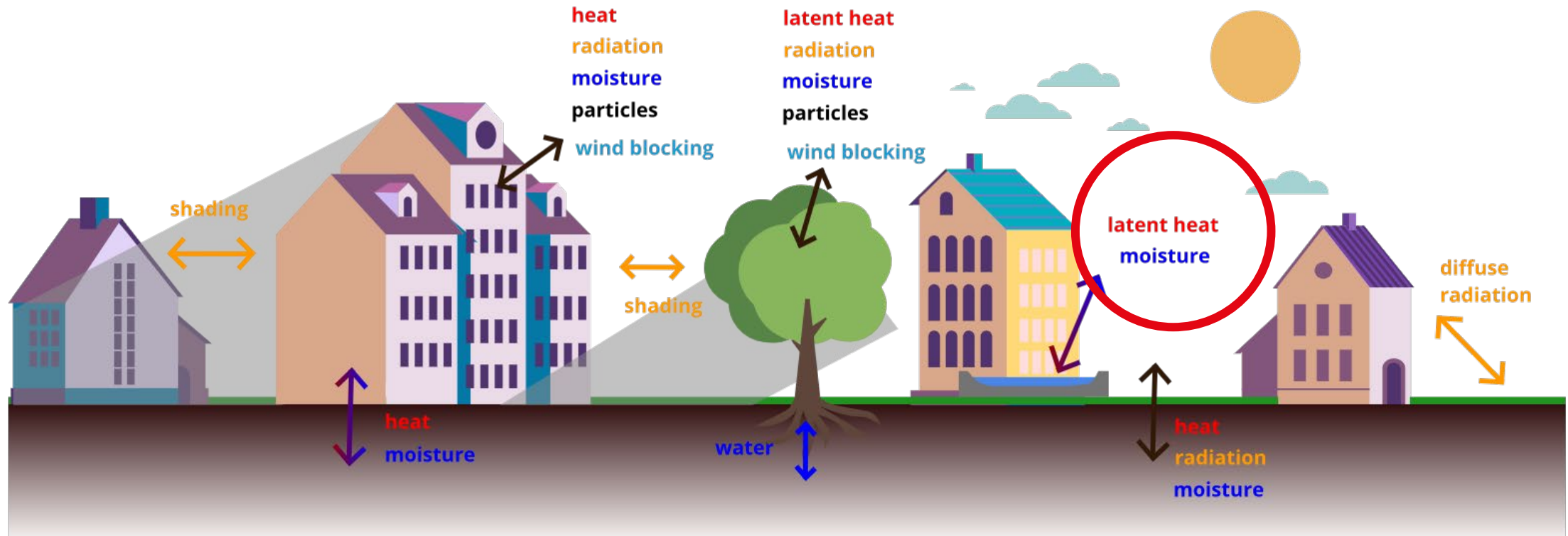
7	24.10		BREAK	
8	31.10	1 x 45'	Urban modeling and computational tools	JY
		1 x 45'	Introduction to the web tool CityTherm (part II)	JY
		1 x 45'	Introduction to the course project II	JY
9	07.11	2 x 45'	Building-environment interaction: thermal, aerodynamic, and hydrodynamic interaction	DK
		1 x 45'	Supervised group work - course project II	JY
10	14.11	2 x 45'	Ground-environment interaction: ground properties, thermal, aerodynamic, and hydrodynamic interaction	DK
		1 x 45'	Supervised group work - course project II	JY
11	21.11	2 x 45'	Water body - environment interaction: thermal, aerodynamic, and hydrodynamic interaction	DK
		1 x 45'	Supervised group work - course project II	JY
12	28.11	2 x 45'	vegetation – environment interaction: characteristics of vegetation, evapotranspiration, aero- and thermal interaction	DK
		1 x 45'	Supervised group work - course project II	JY
13	05.12	2 x 45'	Human Outdoor Comfort: Parameters affecting human comfort and comfort indices (UTCI, PET)	JY
		1 x 45'	Supervised group work - course project II	JY
14	12.12	1 x 45'	Climate-Sensitive Urban Design: complex interaction of all urban elements and their effect on UHI and outdoor comfort	DK
		2 x 45'	Supervised group work - course project II	JY
15	19.12	3 x 45'	Supervised group work on the course project II Course project II submission deadline: 16:00 on December 19	DK, JY



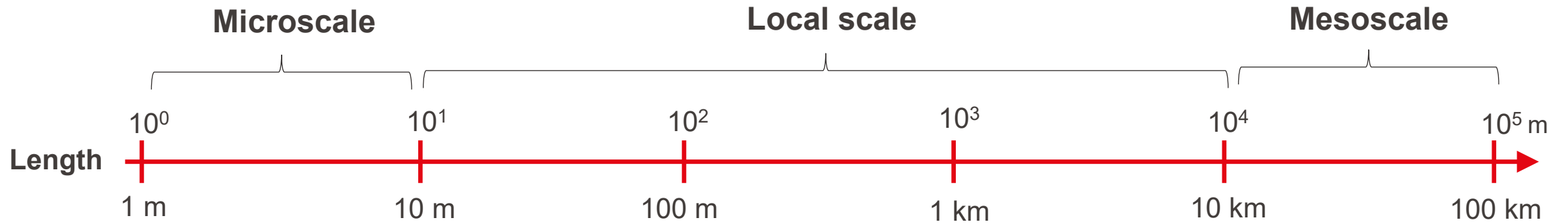
CONTENT:

- I. Introduction
- II. Urban forms of water bodies (lakes, ponds, rivers, fountains, sprays, roof ponds)
- III. Properties of water
- IV. Energy balance of the water bodies
 - Water reservoirs (ponds)
 - Water fountains
 - Water sprays
 - Example exercise

- Blue areas interact *directly* with the atmosphere above. They also have an *indirect* effect on buildings and vegetation.
- Blue areas receive radiation from the Sun. They mainly contribute to the change in atmospheric conditions by the production of **latent heat** and the release of **moisture**.



Reminder from L1



Street canyon



- **Homogeneity vs. heterogeneity** of urban properties (the *smallest* homogeneous urban unit – *an urban block*)
- Spatially, something **homogeneous** at a *one length scale* might be **heterogeneous** at *another* (3D vs 1D treatment of atmospheric properties at the building scale and city scale)

Urban Water Bodies: Introduction

- **Blue areas** or **water bodies** are elements of the urban environment mainly made of water:
 - At the **urban scale**: rivers, sea, and lakes
 - At the **local scale**: ponds, pools, and fountains

- A water body is characterized by its **location, geometry, volume of water** and the **area of free water surfaces in contact with the atmosphere**.

- The *appropriate location* and *form* of water elements significantly affect the **attractiveness** and *strengthening* of the **identity** of places in cities.

- Water bodies *contribute* to the **mitigation of the UHI** by the physical process of **water evaporation**.



Urban Water Bodies: Introduction

- The **atmosphere** over blue bodies is:
 - *Temperate* (usually warmer in winter and cooler in summer) due to the thermal properties of water (its high heat capacity).
 - With *faster wind speeds*, due to the negligible roughness of water surfaces.
- For **cities** exposed to **expanded surfaces of water** bodies, are observed:
 - An *inflow* of *fast, clean* and more *humid* air from these blue areas.
 - A *thermal circulation* of *fresh air* from blue areas that is warmed up over the city and recirculate down to the sea when weather is stable.
- To benefit from these effects, streets should be **wide and open** to the blue areas. On the opposite, if the effects of blue areas are undesirable, buildings should *obstruct* the flow coming from them.
- Water bodies have thermal inertia: peak temperature is reached with a *delay* with respect to peak air temperature.

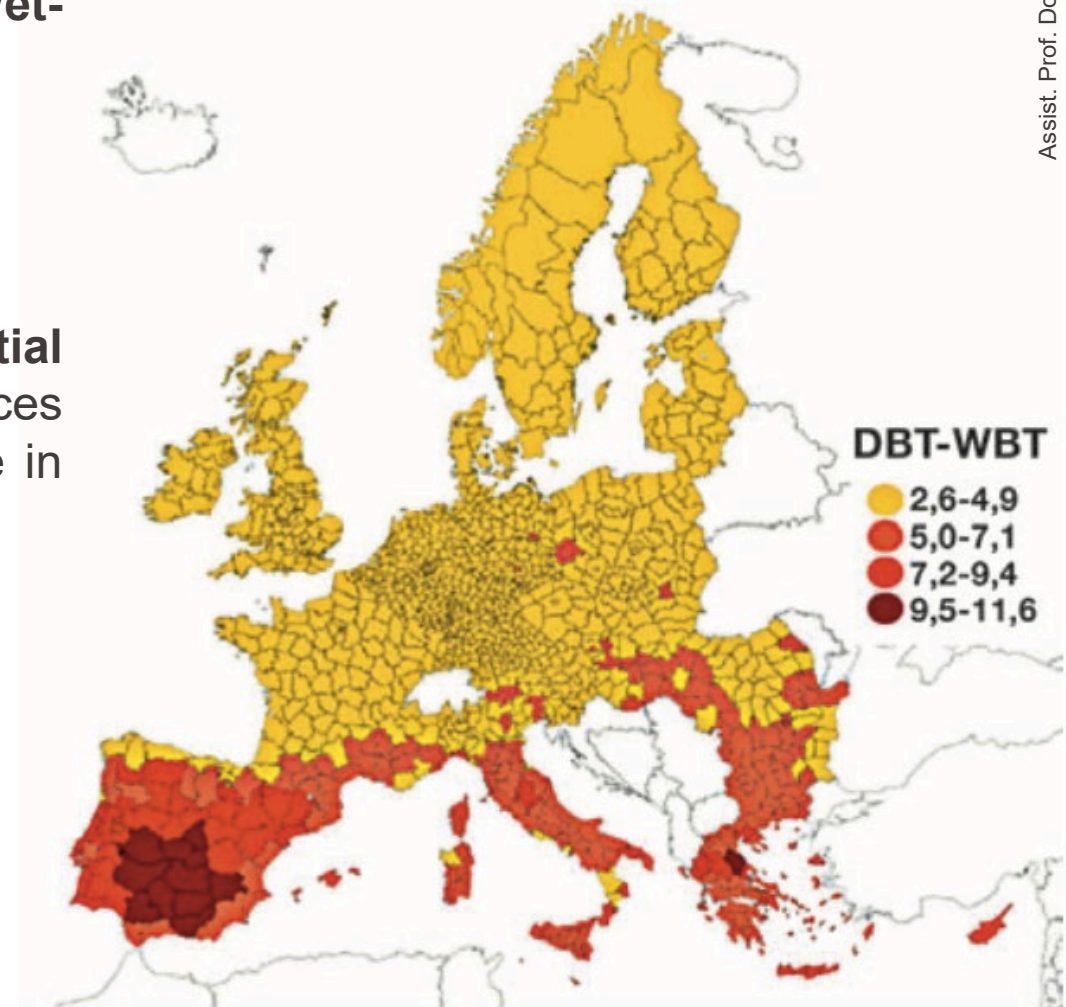
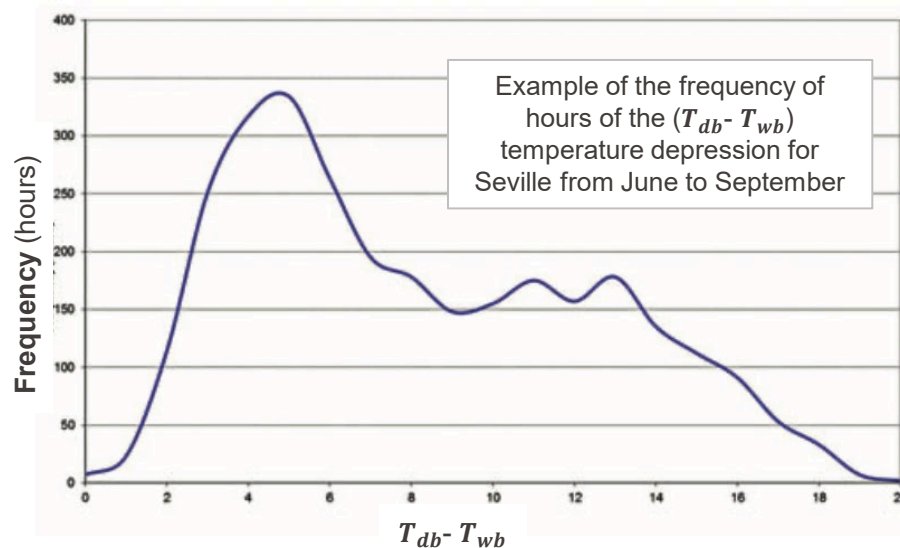


Evaporative cooling: Cooling Degree Days

- The **cooling degree days (CDD)** is the difference between the dry-bulb temperature (T_{db}) and the wet-bulb temperature (T_{wb}) over a day:

$$CDD = \frac{1}{24} \sum_{j=1}^{N \text{ hours}} (T_{db}(i) - T_{wb}(i)) \quad (8-1)$$

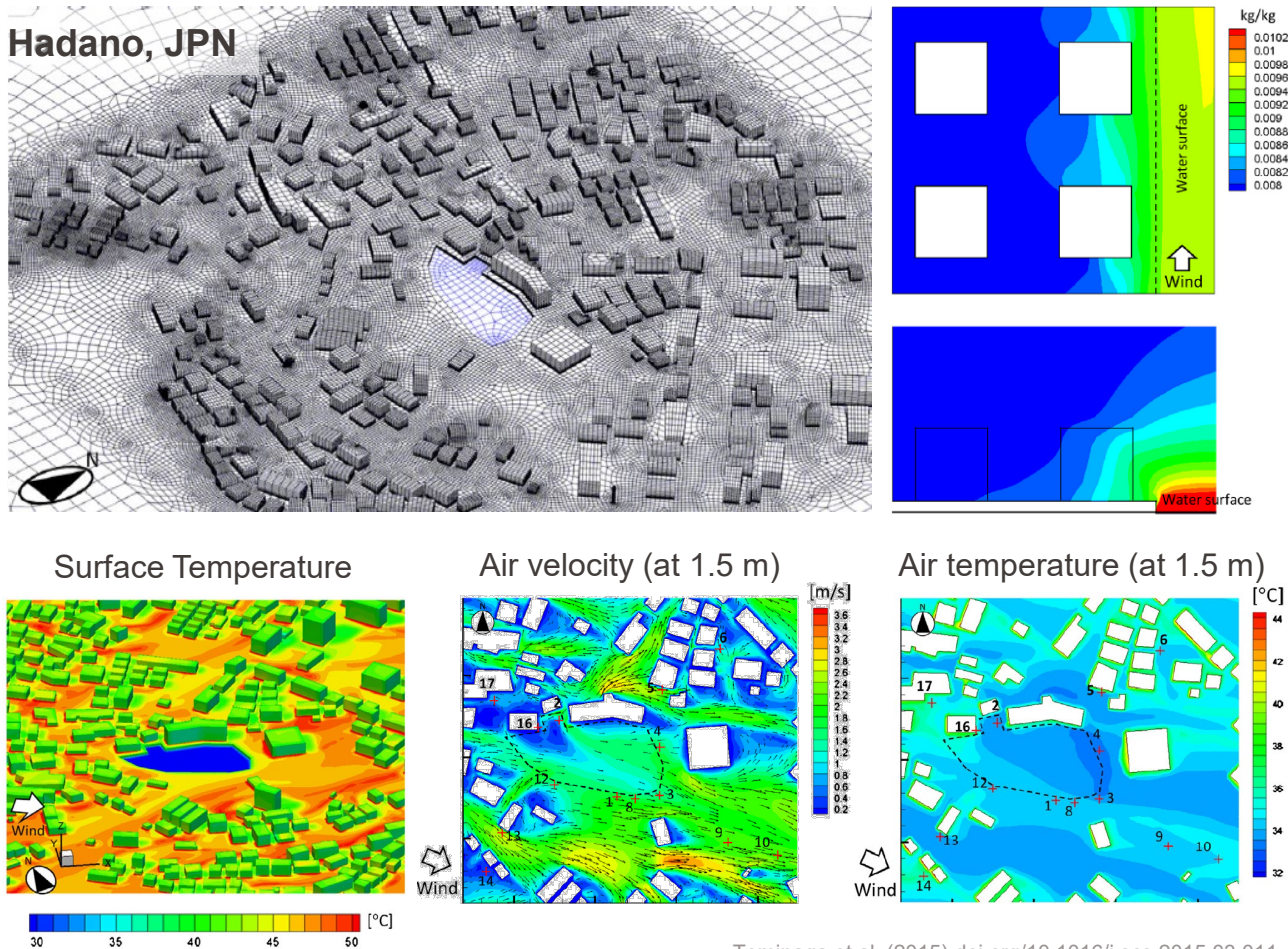
- The **CDD** provides an estimation of the **potential evaporative cooling**. If water is supplied (e.g., surfaces are wetted), the CDD indicates the *average* change in temperature due to evaporation.



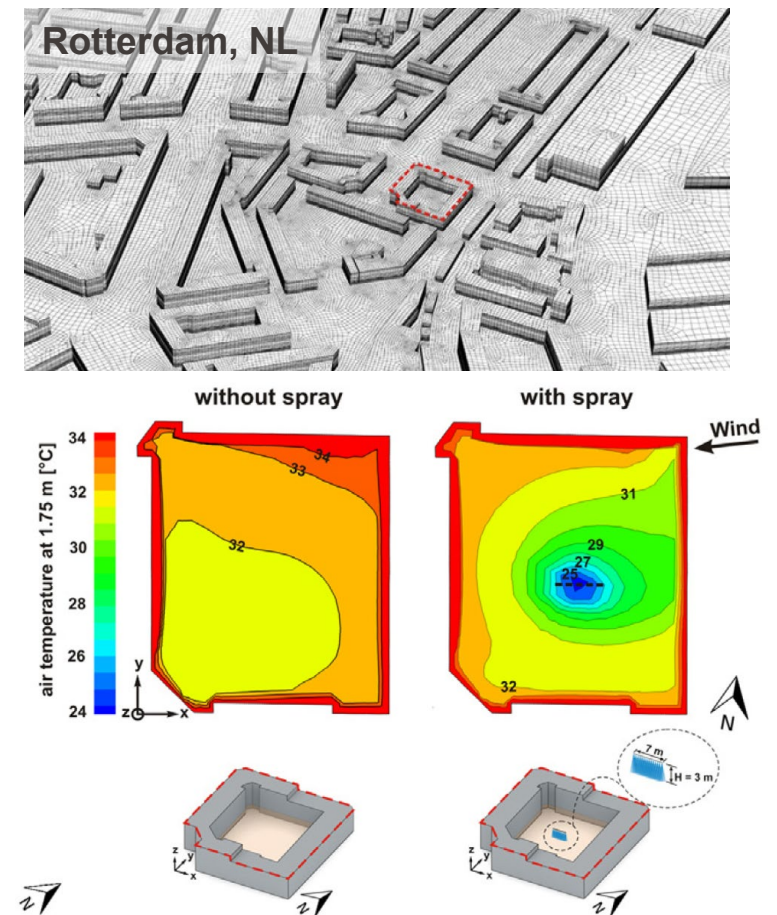
Urban Cooling Islands (UCI)

- Zones inside urban areas with *lower temperatures* due to the presence of vegetation or/and water bodies are referred to as **Urban Cooling Islands (UCI)**.

Cooling effect of the pond



Water spray system

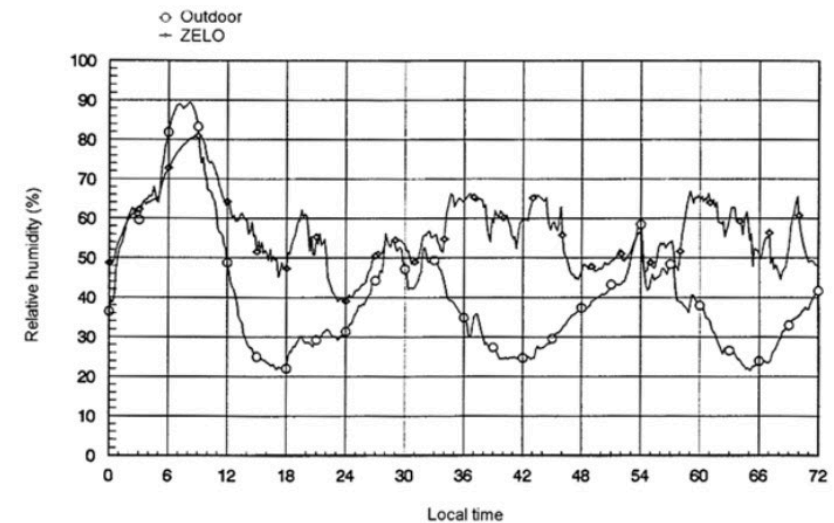
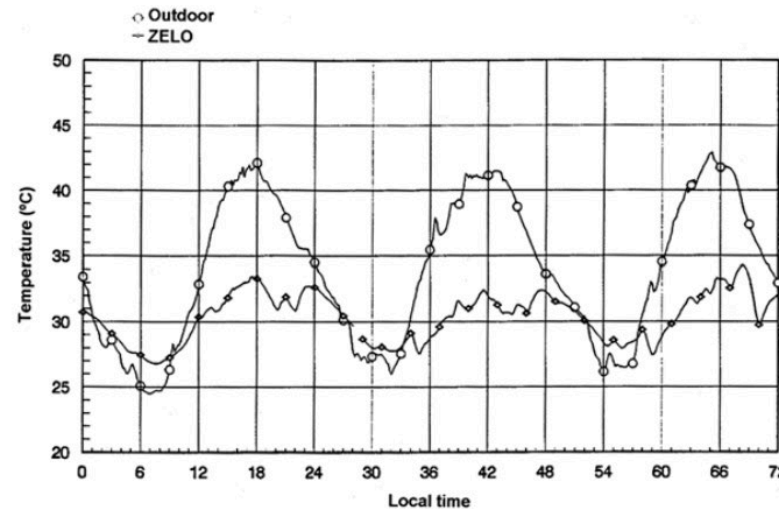


Evaporative cooling: Water sprays

- Droplets of the water sprays evaporate in contact with the surrounding hot air. As cool air is heavier than hot air, a continuous descending flow of cool air is obtained.
- **Water sprays** are more effective when:
 - A droplet radius < 0.5 mm
 - At certain height above the ground for human comfort
 - Nozzles distributed uniformly with no overlap
 - Functioning until *wet-bulb temperature* is reached



- In Sevilla during the Expo'1992, for 128 **micronizers** sheltered with a *continuous flow rate of 7.2 l/h* (ZELO system) were used. A maximum drop of 10 K air temperature was reached and temperature was always maintained lower than outdoor air temperature.



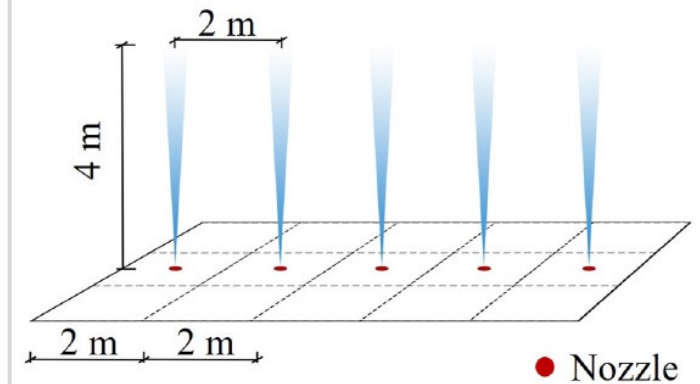
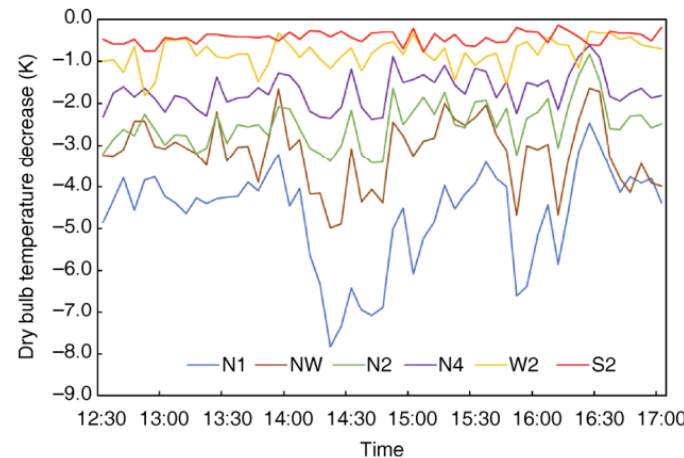
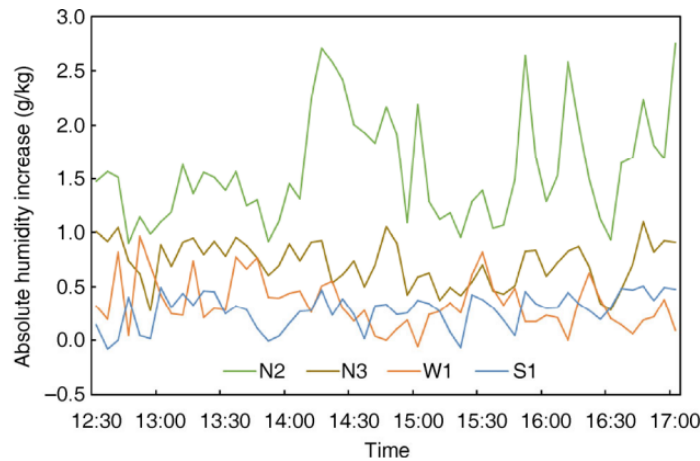
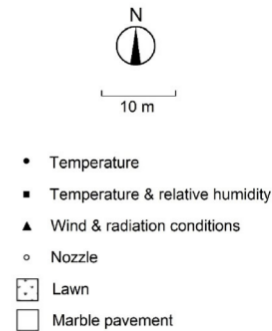
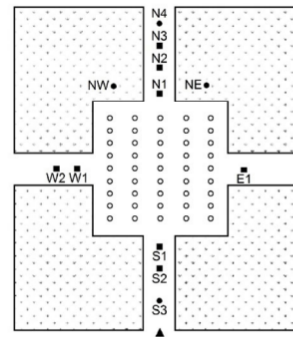
Source: Santamouris and Kolokosta, Urban Climate Mitigation Techniques, p. 124

Evaporative cooling: Fountains

- **Fountains** influence the ambient environment mainly by the interaction (heat, mass and momentum transfer) between the sprayed droplets and air.
- **Cooling** occurs due to convection exchanges between the cooler water and the warmer air and by evaporation.

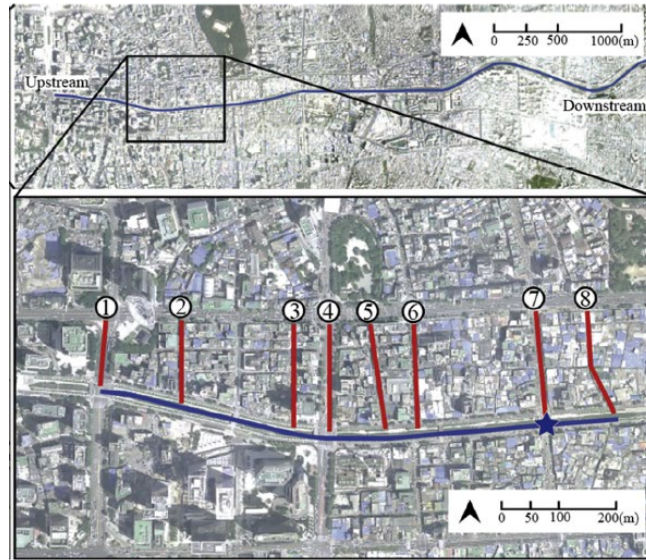


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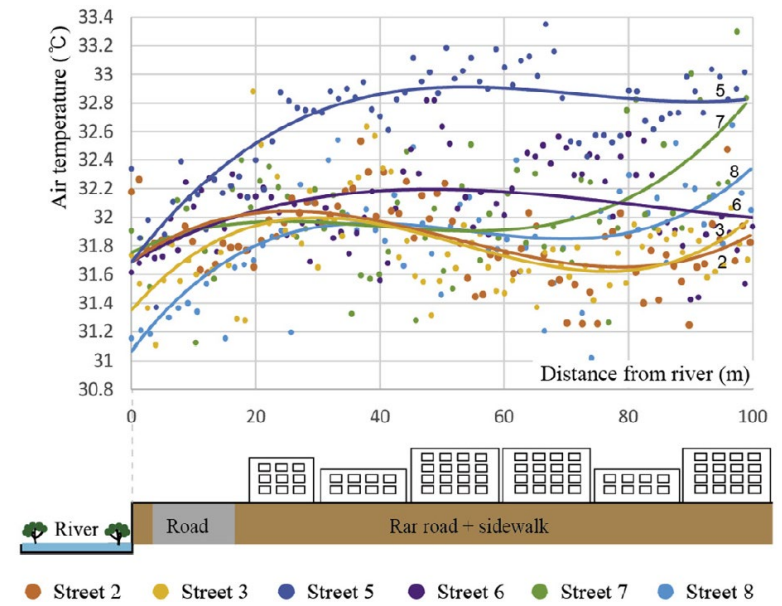
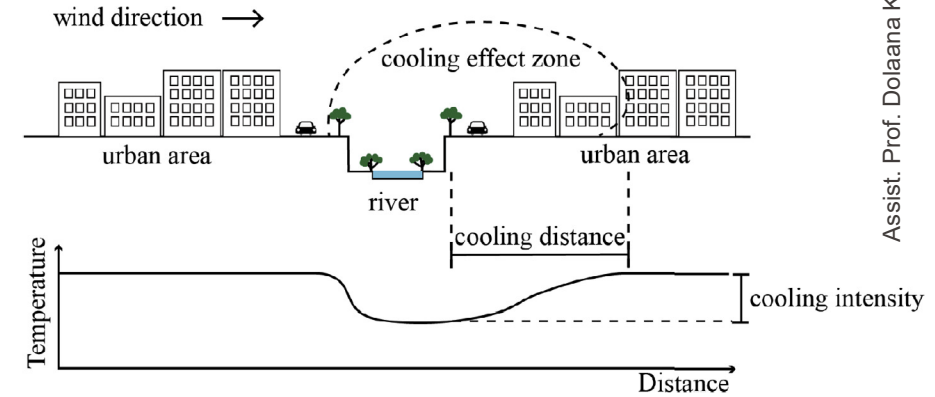


Evaporative cooling: Rivers

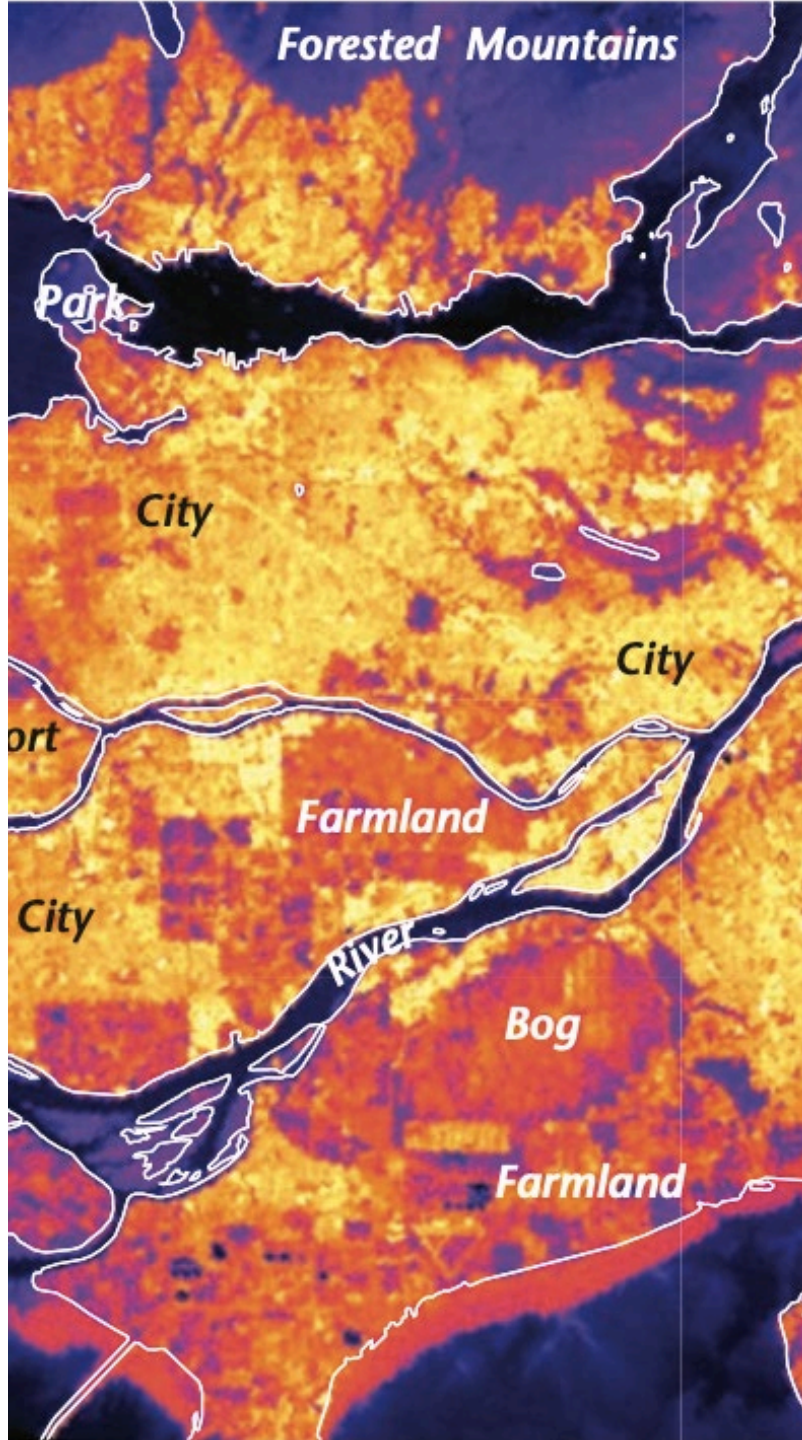
- The effect of **rivers** on microclimate (extent and magnitude) depends on:
 - River width
 - Road and building density of surrounding area
 - Wind speed
- Murakawa et al. (1991) measured an air temperature difference of about 3-5 °C between the river and the city for wind speeds of 1-5 m/s.



Seoul, KR, Cheonggye River



Park et al. (2019) doi.org/10.1016/j.landurbplan.2018.10.022



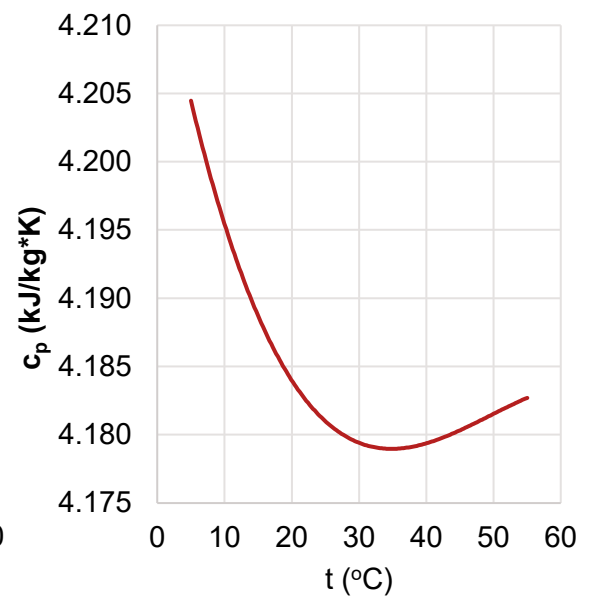
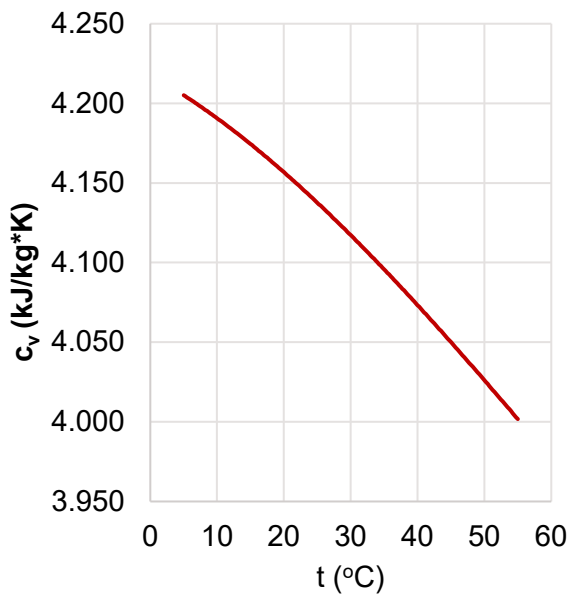
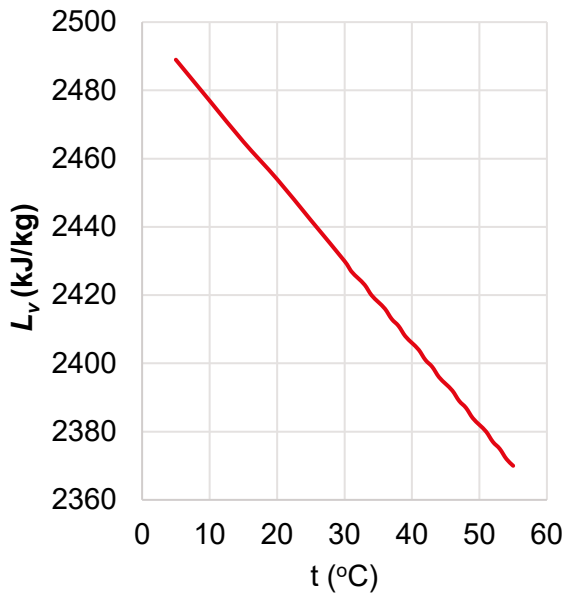
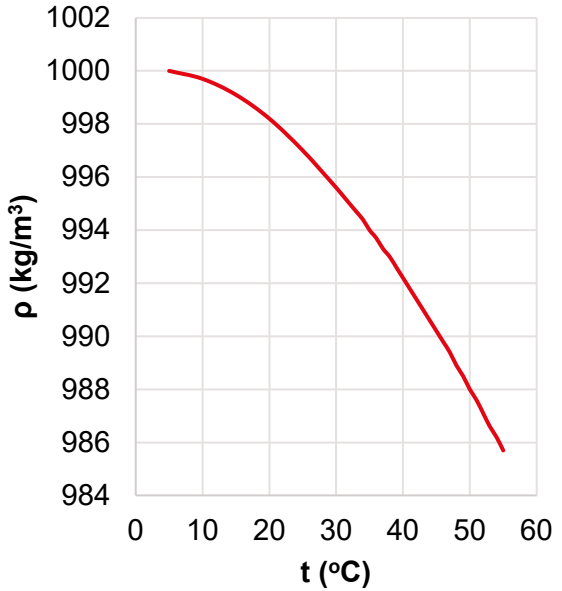
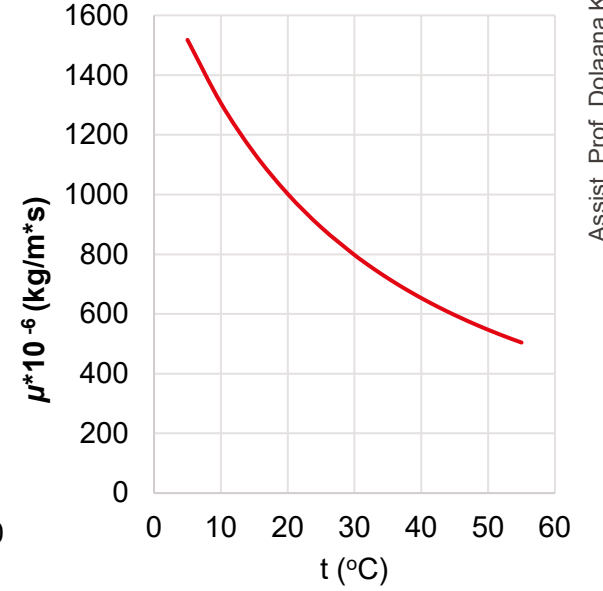
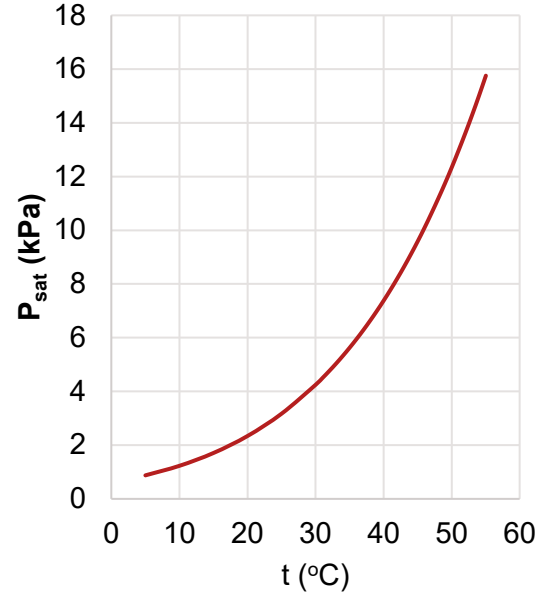
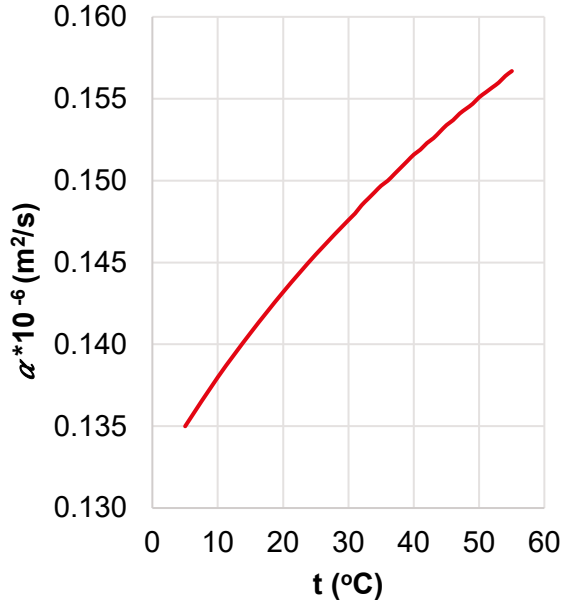
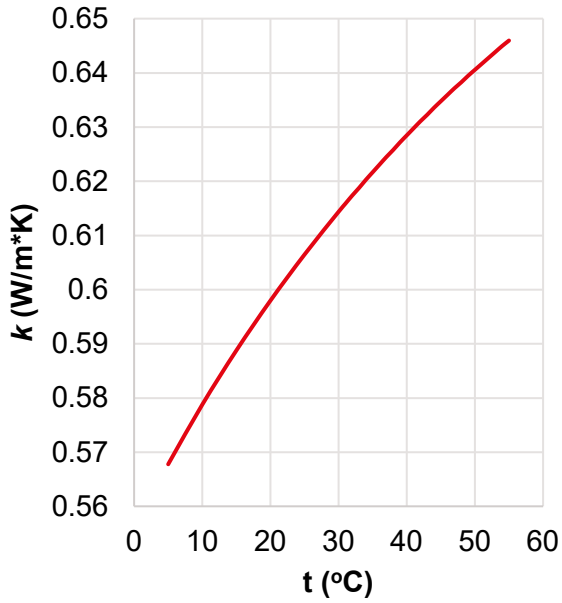
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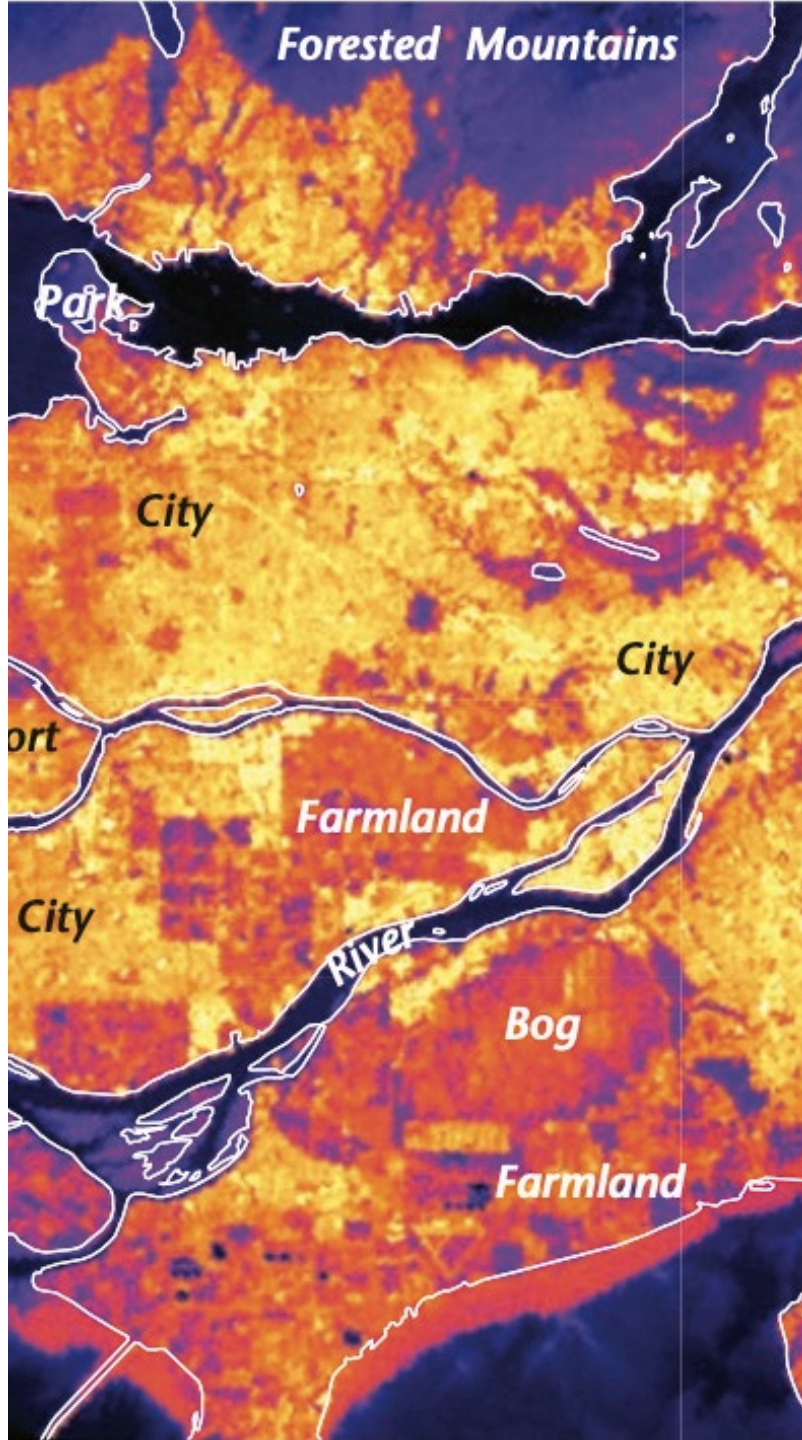
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Material	State	Heat capacity C ($\text{MJ m}^{-3} \text{K}^{-1}$)	Thermal conductivity k ($\text{W m}^{-1} \text{K}^{-1}$)	Thermal diffusivity κ ($\text{m}^2 \text{s}^{-1} \times 10^{-6}$)	Thermal admittance μ_s ($\text{J m}^{-2} \text{s}^{-1/2} \text{K}^{-1}$)
Construction and building materials in dry state (built sites)					
Asphalt road	Range	1.92–2.10	0.74–1.40	0.38–1.04	1,205–1,960
	Typical	1.94	0.75	0.38	1,205
Concrete	Aerated	0.28	0.08	0.29	150
	Dense	2.11	1.51	0.72	1,785
Stone	Typical	2.25	2.19	0.97	2,220
Brick	Typical	1.37	0.83	0.61	1,065
Natural materials (rural and undeveloped urban sites)					
Sandy soil (40% porosity)	Dry	1.28	0.3	0.24	620
	Saturated	2.96	2.2	0.74	2,550
Clay soil (40% porosity)	Dry	1.42	0.25	0.18	600
	Saturated	3.10	1.58	0.51	2,210
Peat soil (80% porosity)	Dry	0.58	0.06	0.10	190
	Saturated	4.02	0.5	0.12	1,420
Water¹	4°C, still	4.18	0.57	0.14	1,545
Air¹	10°C, still	0.0012	0.025	21.5	5
	Turbulent	0.0012	~125	~10 × 10 ⁶	390

¹ Properties depend on temperature

Water: Thermo-Physical Properties





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Energy Balance of Water Reservoirs

- Blue areas interact with the atmosphere, buildings and other urban elements *somewhat similarly* to ground and vegetation.
- The water bodies interaction with their environment is always *at balance*. It is the sum of radiation, sensible heat, latent heat, ground heat and stored heat.

$$Q^* = Q_H + Q_E + Q_G + \Delta Q_S \quad (1-3a)$$

Radiation budget

Sensible heat

Latent heat

Ground heat

Stored heat

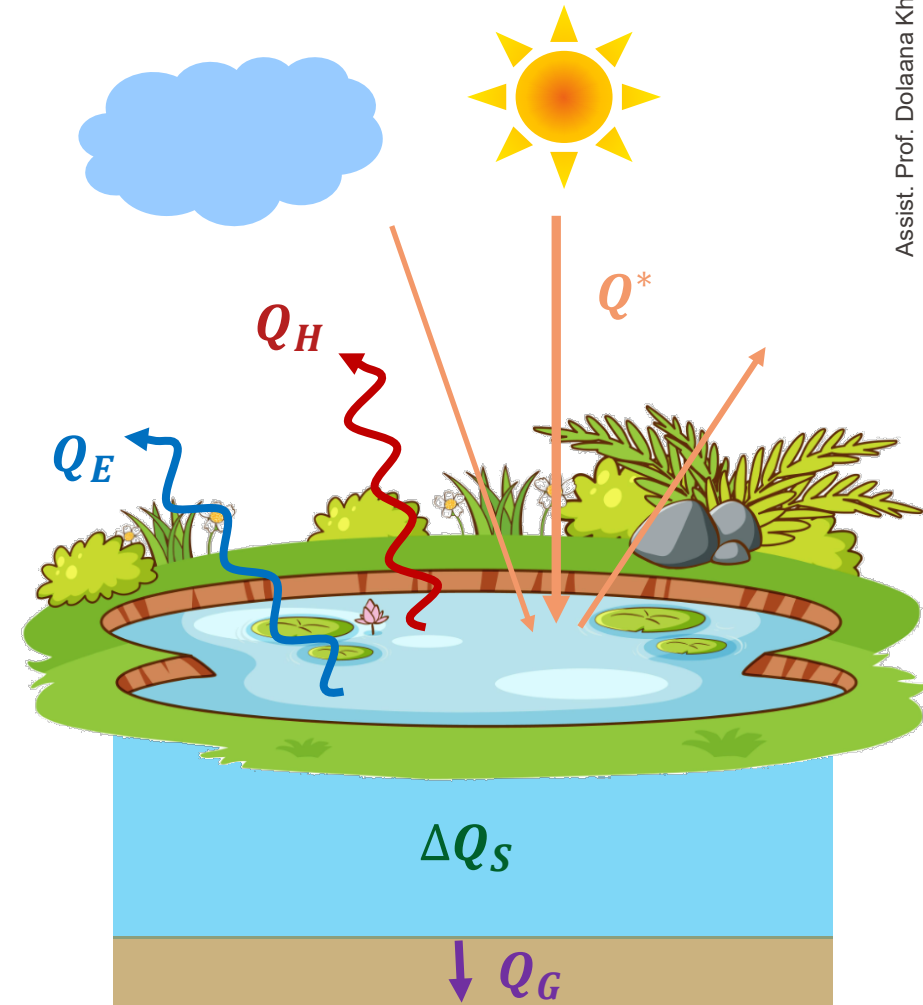
Q^* - **net allwave radiation heat flux** (W/m^2)

Q_H - **sensible heat flux** (W/m^2)

Q_E - **latent heat flux** (W/m^2)

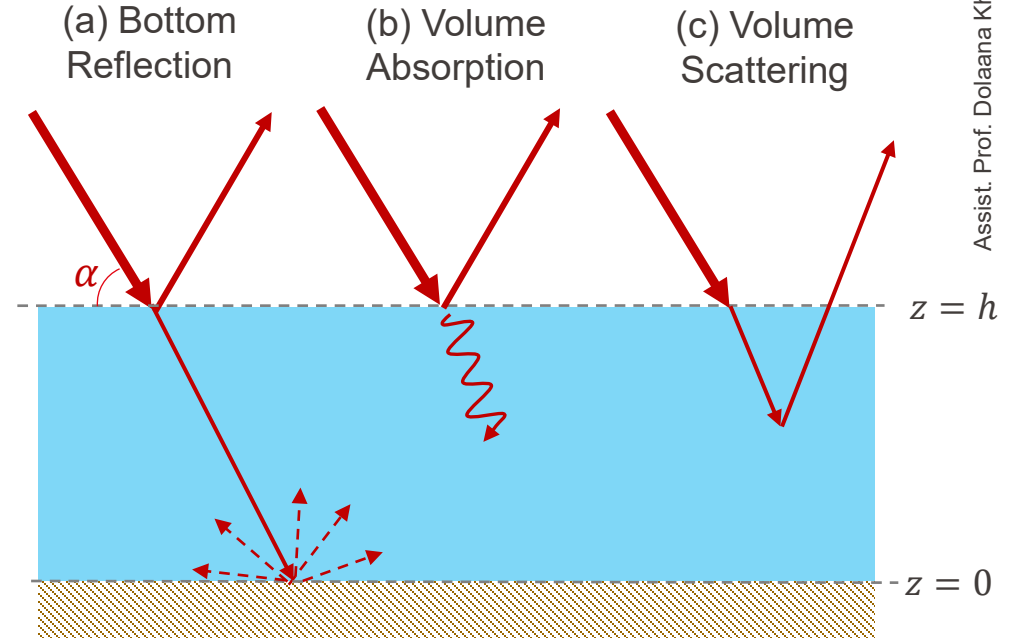
Q_G - **ground heat flux** (sensible heat by conduction to the substrate) (W/m^2)

ΔQ_S - **stored heat in the water body** (W/m^2)



Water Reservoirs: Radiation Budget

- The **radiation balance** of a water body is not straightforward as radiation can penetrate to *considerable depth* in water. Thus, radiation exchange not only happens on the *water surface* but also within *the volume of water*.
- **Radiation entering** a water body can *interact* with it in several different ways depending on the **absorption** and **backscattering processes** within a water body mainly caused by the water molecules themselves and by dissolved or suspended substances.



- **Case (a), clear water with bottom reflection**

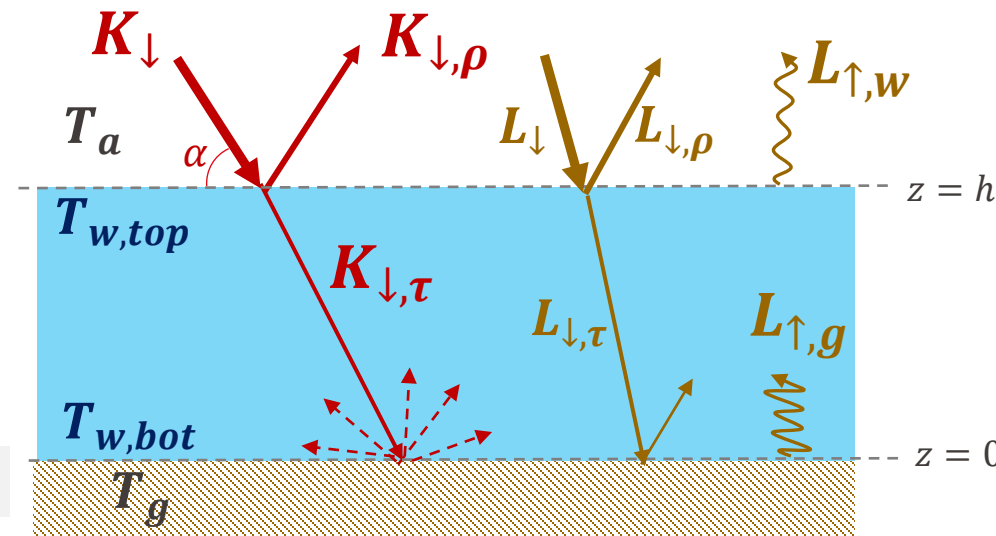
- Radiation budget on the **top surface** ($z = h$):

$$Q^* = (K_{\downarrow} - K_{\downarrow,\rho}) + (L_{\downarrow} - L_{\uparrow}) \quad (3-37)$$

$$(8-2) \quad Q_{top}^* = (K_{\downarrow} - \rho \cdot K_{\downarrow}) + (L_{\downarrow} - \rho \cdot L_{\downarrow} - \varepsilon \cdot \sigma \cdot T_{w,top}^4)$$

- Radiation budget on the **bottom surface** ($z = 0$):

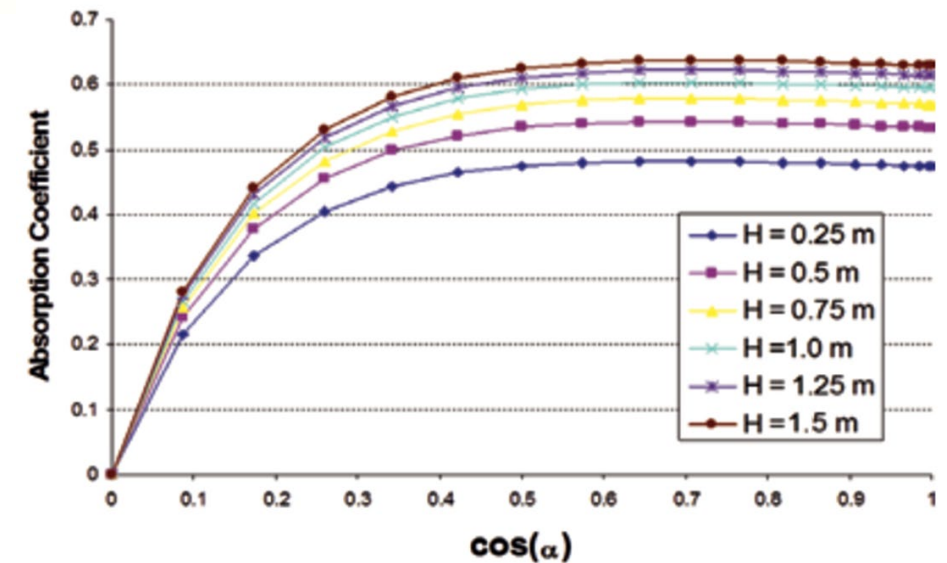
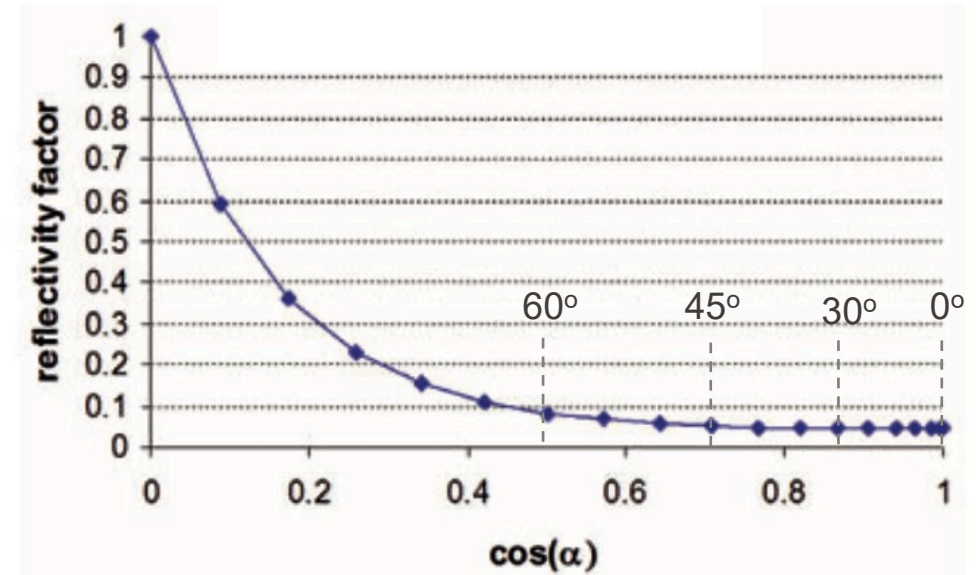
$$(8-3) \quad Q_{bot}^* = (K_{\downarrow,\tau} - \rho_g \cdot K_{\downarrow,\tau}) + (L_{\downarrow,\tau} - \rho_g \cdot L_{\downarrow,\tau} - \varepsilon_g \cdot \sigma \cdot T_g^4)$$



Water: Radiative Properties

- Radiative properties of water (absorptivity α , reflectivity ρ , and transmittance τ) depend on **the angle of incidence of the light** and **the water depth** (path length for solar radiation):
 - Reflectivity *only* depends on the solar incident angle. The water has reflectivity < 0.1 when incoming radiation angle $\alpha < 65^\circ$; the *minimum* reflectivity of **0.05** at zenith ($\alpha = 0^\circ$, $\cos(\alpha) = 1$).
 - Absorptivity depends on the solar incident angle α and the pond depth H ; the percentage of radiation absorbed increases with the water depth.
 - Generally, water is transparent to short-wave radiation, however, absorption depends not only on the path length but also strongly on the **turbidity** of the water

- The **low reflectivity of water** is an **advantage to lower the diffuse radiation** perceived by *urban objects*.



Source: Santamouris and Kolokosta, Urban Climate Mitigation Techniques, p. 124

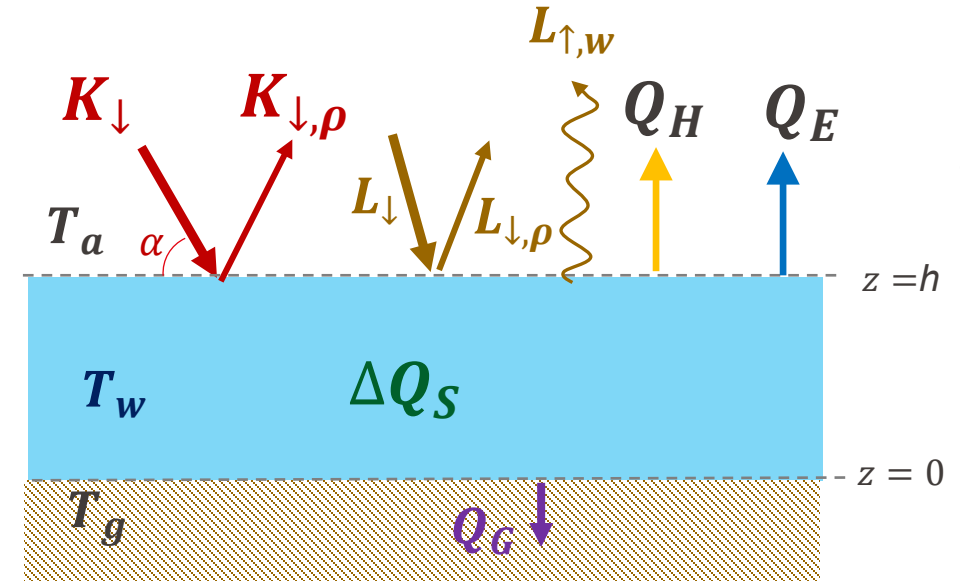
Water Reservoirs: Thermal Storage

- The energy balance** of the water body can be expressed for **top** and **bottom surfaces**, as well as for **the water mass**. For shallow bodies, such as artificial water fountains, water temperature could be considered as *uniform*, only depend on time.
- Radiation* absorbed by water *raises* its temperature. **Thermal inertia** due to **the storage capacity of water** has the advantage to **delay** and **buffer** the temperature increase.
- Stored heat ΔQ_S** is equal to *the difference between the radiation absorbed by the pond volume with the heat losses at the bottom and the top of the pond*:

$$\Delta Q_S = (K_{\downarrow,\tau} + L_{\downarrow,\tau}) - (L_{\uparrow,w} + Q_H + Q_E + Q_G) \quad (8-4)$$

- Stored heat ΔQ_S** is expressed as a function of the specific heat capacity of water and temperature gradient:

$$\Delta Q_S = \rho \cdot c_p \cdot h \cdot \frac{dT_w}{dt} \quad (8-5)$$

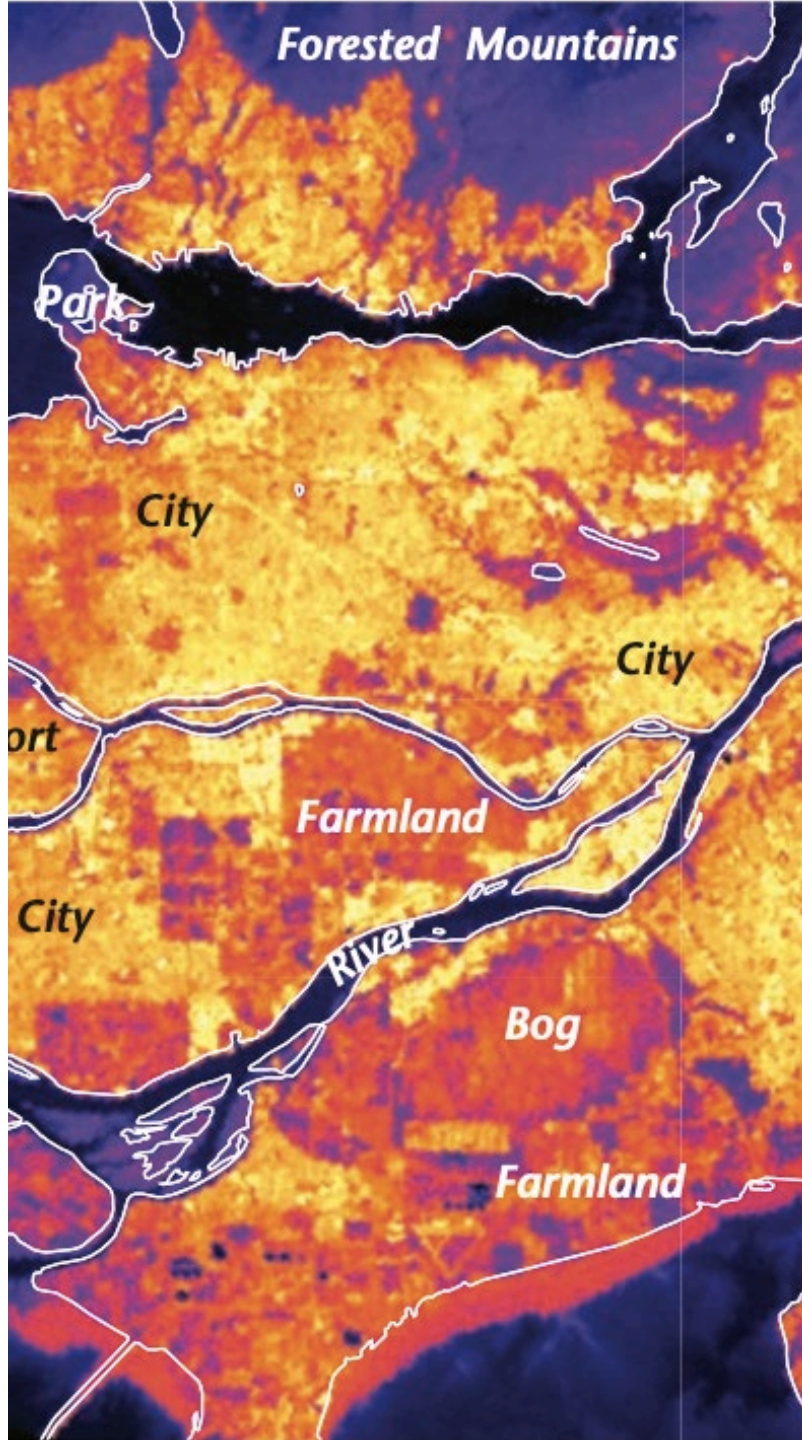


$K_{\uparrow,\tau} = (K_{\downarrow} - K_{\downarrow,\rho})$ - absorbed shortwave radiation (direct and diffuse)

$L_{\downarrow,\tau} = (L_{\downarrow} - L_{\downarrow,\rho})$ - absorbed longwave radiation from the environment

- Sensible heat flux Q_G** by conduction to the substrate:

$$Q_G = h_{conv,w} \cdot (T_w - T_g) \quad (8-6)$$



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Water Fountains: Jet Breakup

- **Fountains** release water into the air in various patterns for decorative or practical purposes.
- After the water has been sprayed out, it becomes a *water column* near the nozzle. The **jet breakup process** determines the *initial conditions* of all the sprayed droplets:
 - **Primary breakup** - the droplets are discomposed from the water column
 - **Secondary breakup** - If the droplet diameter is larger than *the critical value*, it will continue splitting into smaller droplets.
- The **jet breakup length** l (i.e. the distance between the onset of breakup and the nozzle) is dependent on the nozzle diameter:

$$l = 8.51 \cdot D \cdot We_{LD}^{0.32}$$

where

$$We_{LD} = \frac{\rho_w \cdot u_{0,mean}^2 \cdot D}{\sigma}$$

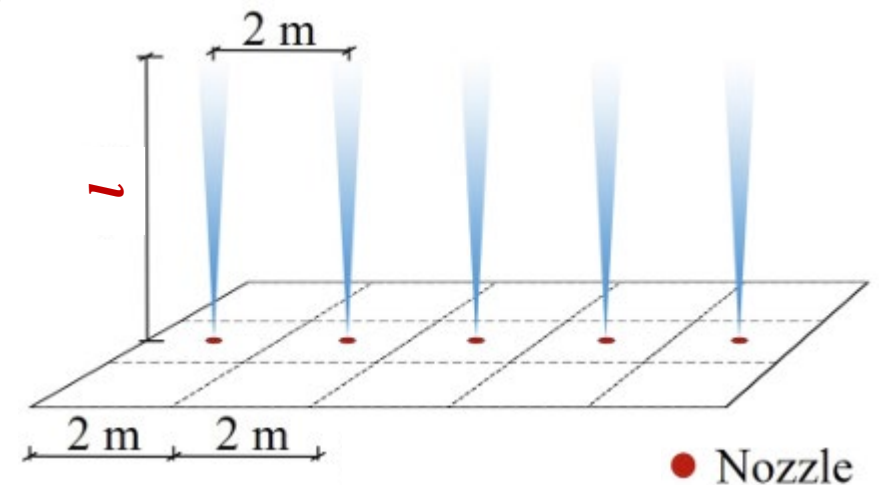
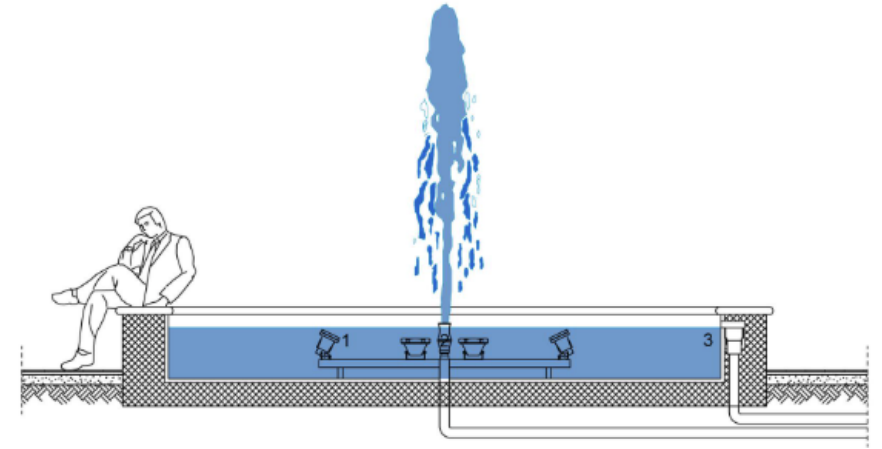
- D – nozzle diameter (m)

- We_{LD} – jet exit Weber number (-)

- ρ_w – water density (kg/m³)

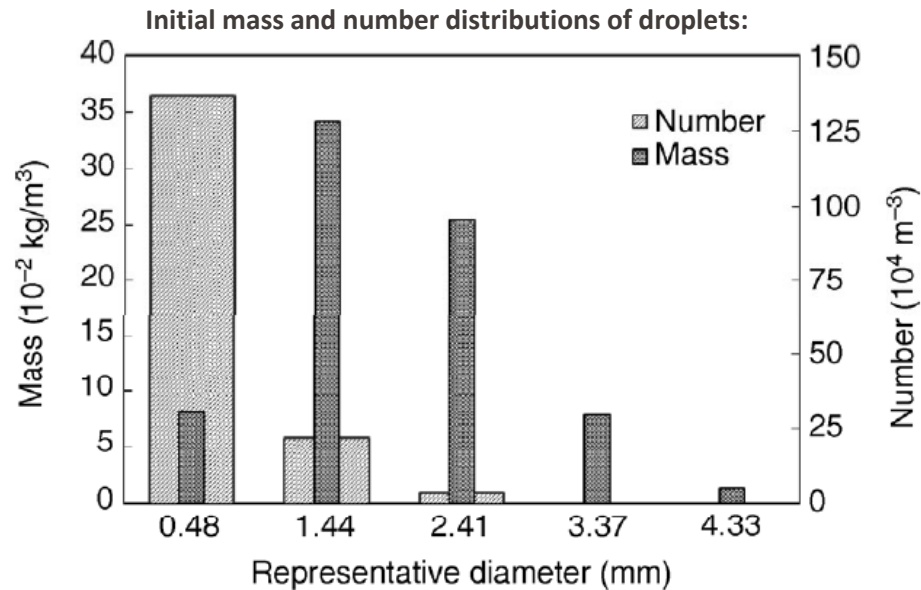
- $u_{0,mean}$ – jet exit mean velocity (m/s)

- σ – surface tension of water (N/m)

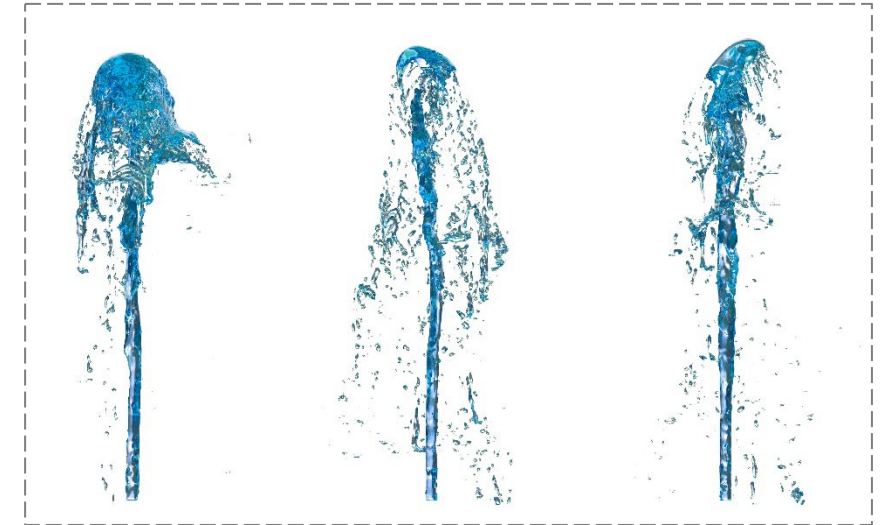


Water Fountains: Water Droplets Distribution

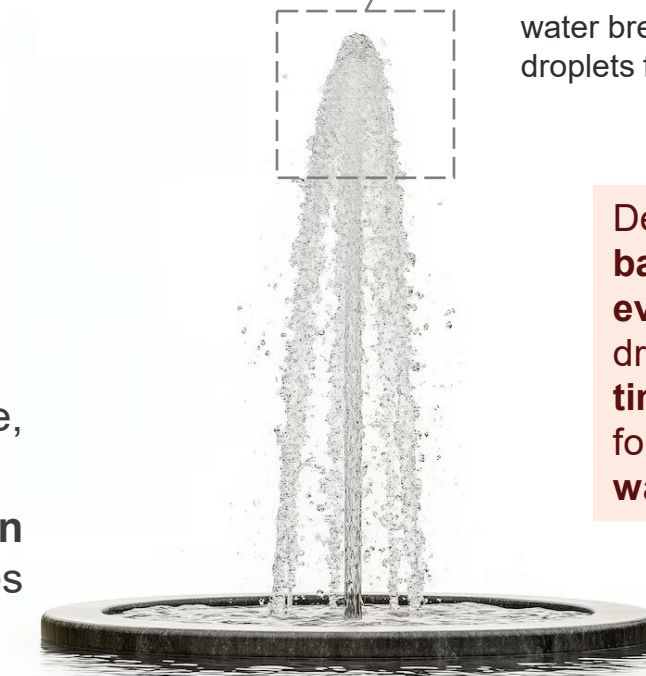
- Fountain produce droplets of different sizes. The **distribution of sizes** is *not uniform* typically the number of droplets decreases as size increases.
 - *Example:* a vertical water jet with the jet velocity $u_{0,mean}$ of 9.9 m/s



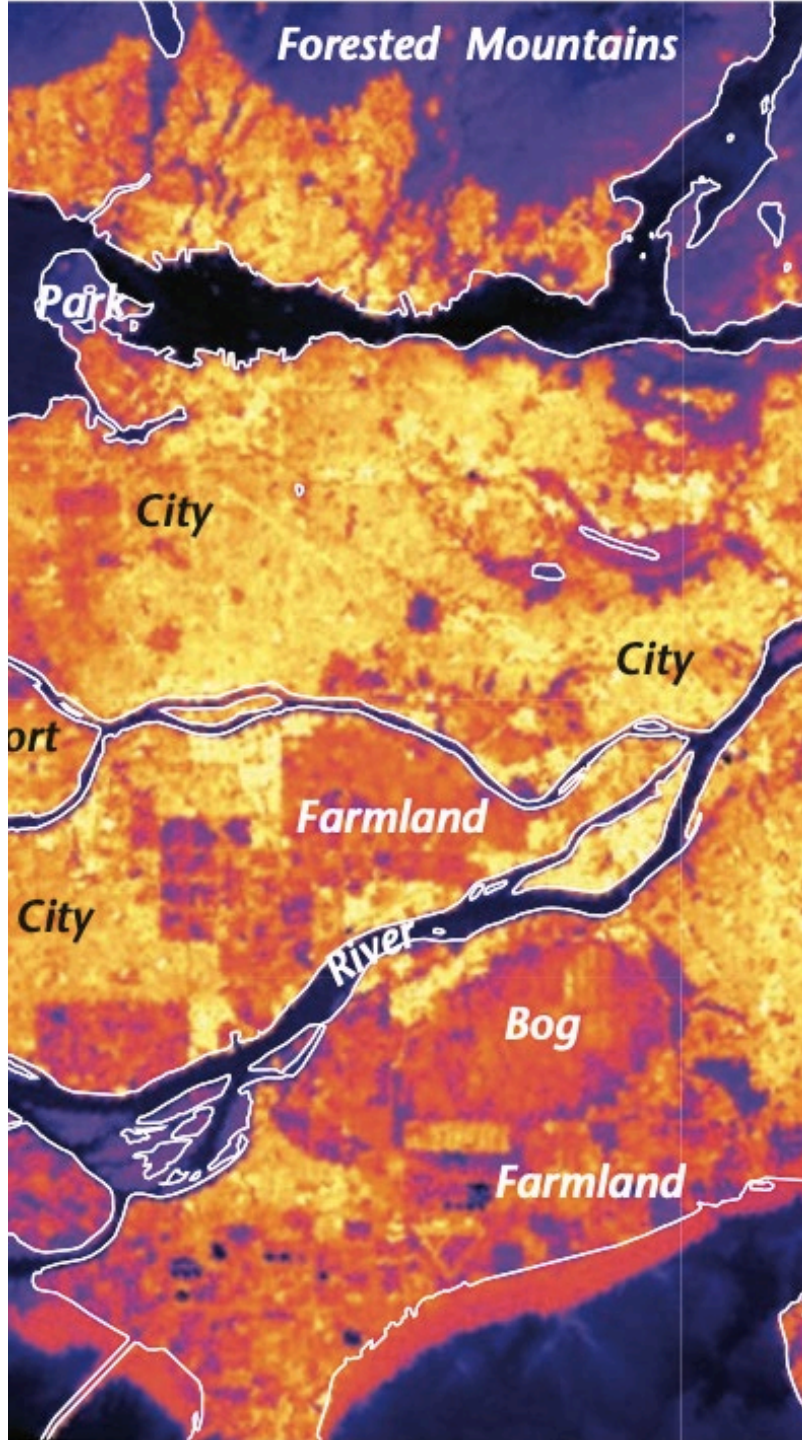
- **Factors affecting the distribution of droplets:** water pressure, nozzle type, and fluid properties (viscosity, surface tension).
- Because the droplets have *different masses*, **they remain airborne for different lengths of time**; the smaller ones evaporate *before* they can fall back to the ground



Different patterns of water breakup and droplets formation



Details on the **energy balance** and **evaporation** of water droplets, their **travel time** are in the following section on **water sprays**.



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Evaporation rate: Transport of Water Vapor

Reminder from L4

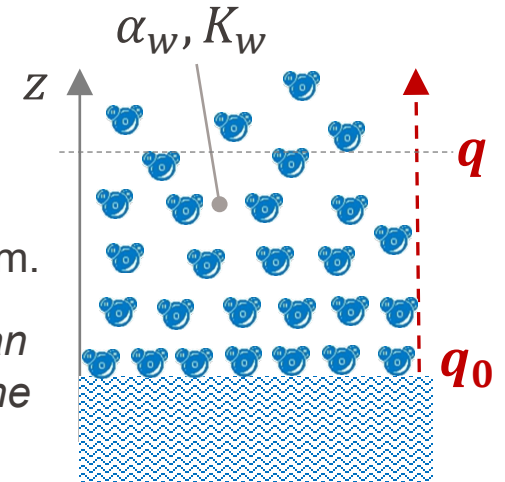
- The expression of the **evaporation rate** E ($\frac{kg}{m^2s}$) due to the **transport of water vapor** over a **surface** depends whether the **flow** is **laminar** or **turbulent**.

- Laminar flow: Fick's law** for the evaporation rate at a *horizontal surface*:

$$E = -\rho \cdot \alpha_w \cdot \frac{\partial q}{\partial z} \quad (4-17)$$

α_w (m^2/s) - molecular diffusivity of water vapor
 q (kg/kg) - specific humidity (see L2, slide 17)
 ρ (kg/m^3) - density of water vapor mixture

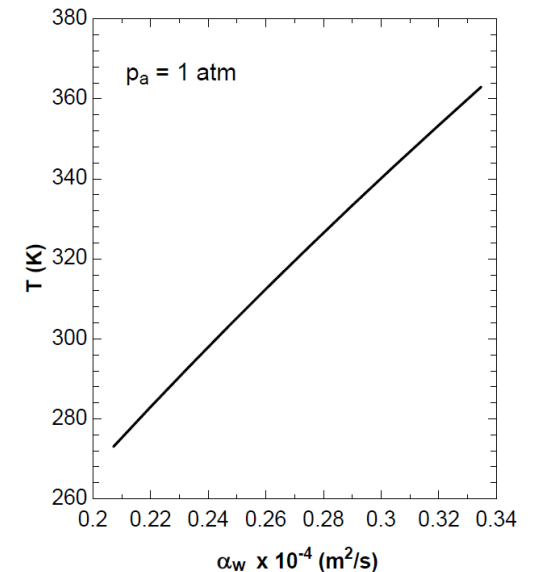
- Only valid when molecular exchange is the primary (perhaps the only) transport mechanism.
- Fick's law is not typically applicable for *urban areas* because the flow in the atmosphere at the *surface layer* at local scale is *always turbulent*.
- It is applicable for mass transfer of *water droplets* as flow around them is mostly laminar



- Molecular diffusivity of water vapor** α_w (m^2/s):

$$\alpha_w = 1.97 \times 10^{-5} \cdot \left(\frac{p_0}{p_a}\right) \cdot \left(\frac{T}{T_0}\right)^{1.685} \quad (8-7)$$

- $p_0 = 1 \text{ atm}$, $T_0 = 256 \text{ K}$
- validity: T range of 273-373 K
- p_a (atm) - atmospheric pressure,
- T (K) - film temperature $(T_0 + T)/2$



Urban Water Sprays

- **Urban water sprays (misting systems)** are public infrastructure systems that release fine mist or water droplets into the air in city environment.
- They use **high-pressure nozzles** to create **microscopic water droplets**. These droplets **evaporate quickly**, *absorbing* heat from the surrounding air.
- The main purpose is **to cool public spaces** and **to mitigate the UHI effect**.



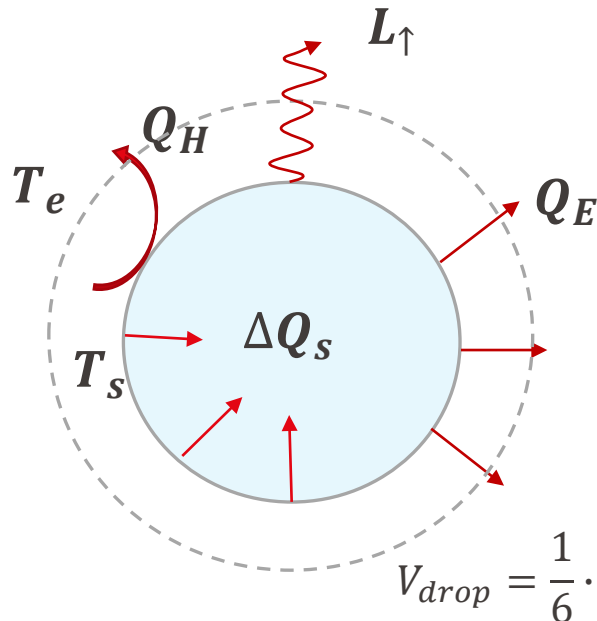
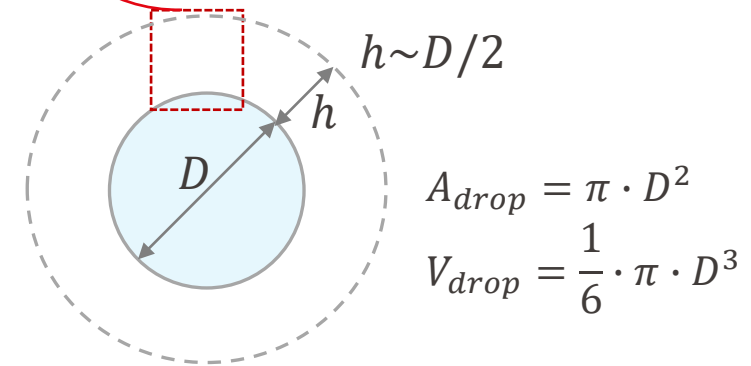
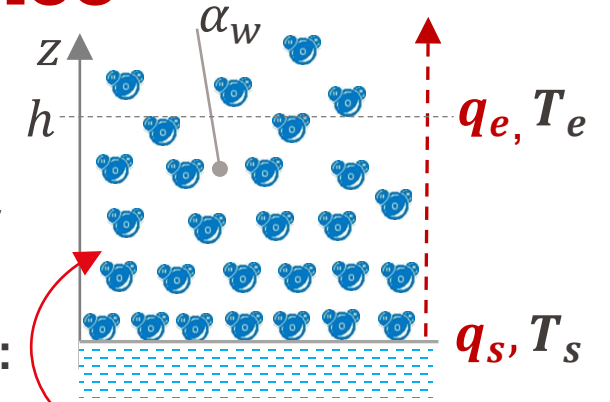
EPFL Water Sprays: Evaporation & Energy Balance

- Assumptions:
 - A water drop is considered to be a sphere
 - No interaction with among the droplets
 - The whole volume of the water drop is at the temperature T_s
 - Radiation heat transfer is negligible
- A **mass balance** on a droplet (mass loss equal to the evaporation rate):

$$\frac{d(V_{drop} \cdot \rho_w)}{dt} = -E \cdot A_{drop} \quad (8-8)$$

- Evaporation rate of the droplet (per Fick's law), in $\frac{kg}{m^2s}$:

$$E = \rho \cdot \alpha_w \cdot \frac{(q_s - q_e)}{h} \quad (4-17)$$



- Energy balance (in W):** the rate of *decrease* in droplet internal energy (energy storage ΔQ_s) is *mainly* balanced by the rate of latent heat supply (Q_H) and sensible heat transfer (Q_E); radiation heat transfer L_{\uparrow} relatively is *small*.

$$-\Delta Q_s = Q_H + Q_E \quad (8-9)$$

$$-\rho_w \cdot \left(\frac{1}{6} \cdot \pi \cdot D^3\right) \cdot c_v \cdot \frac{dT}{dt} = \pi \cdot D^2 [h_{conv} \cdot (T_s - T_e) + L_v \cdot E] \quad (8-10)$$

EPFL Water Sprays: Droplet Lifetime

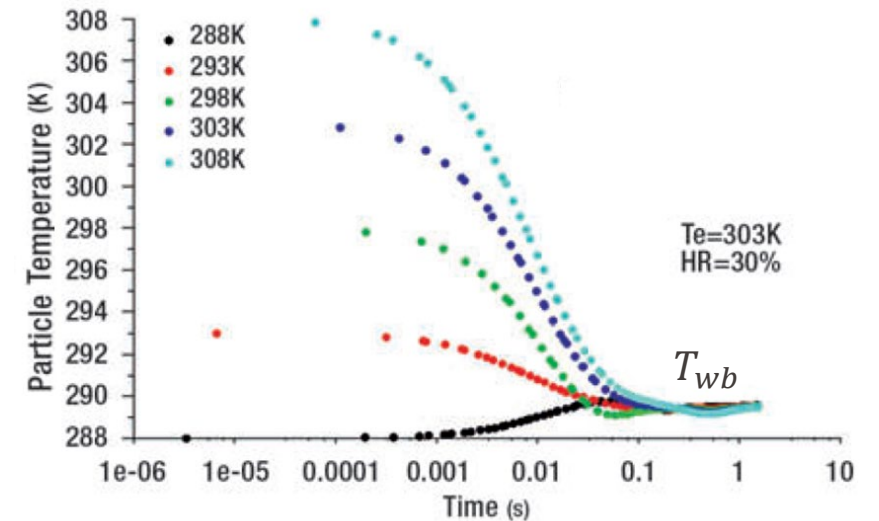
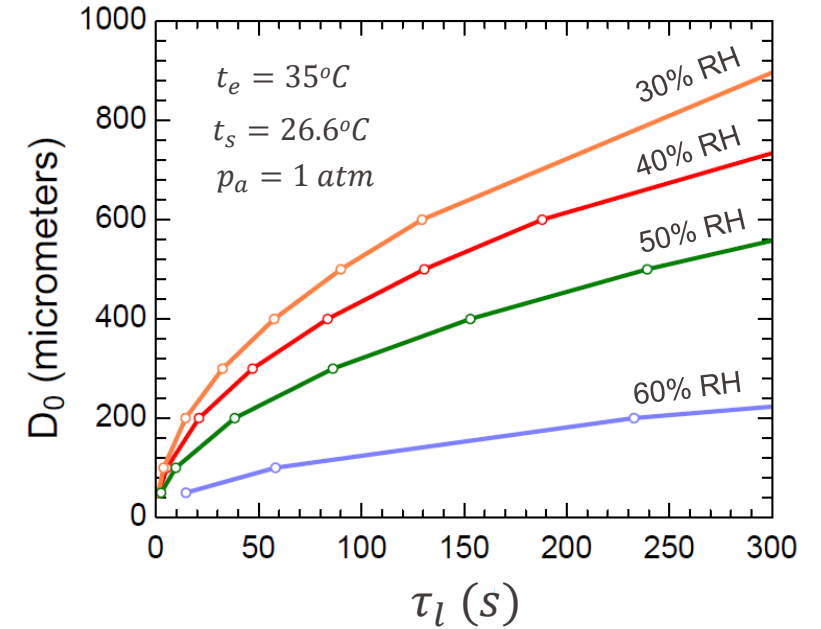
- The droplet lifetime τ_l (s) after injection into an air stream:

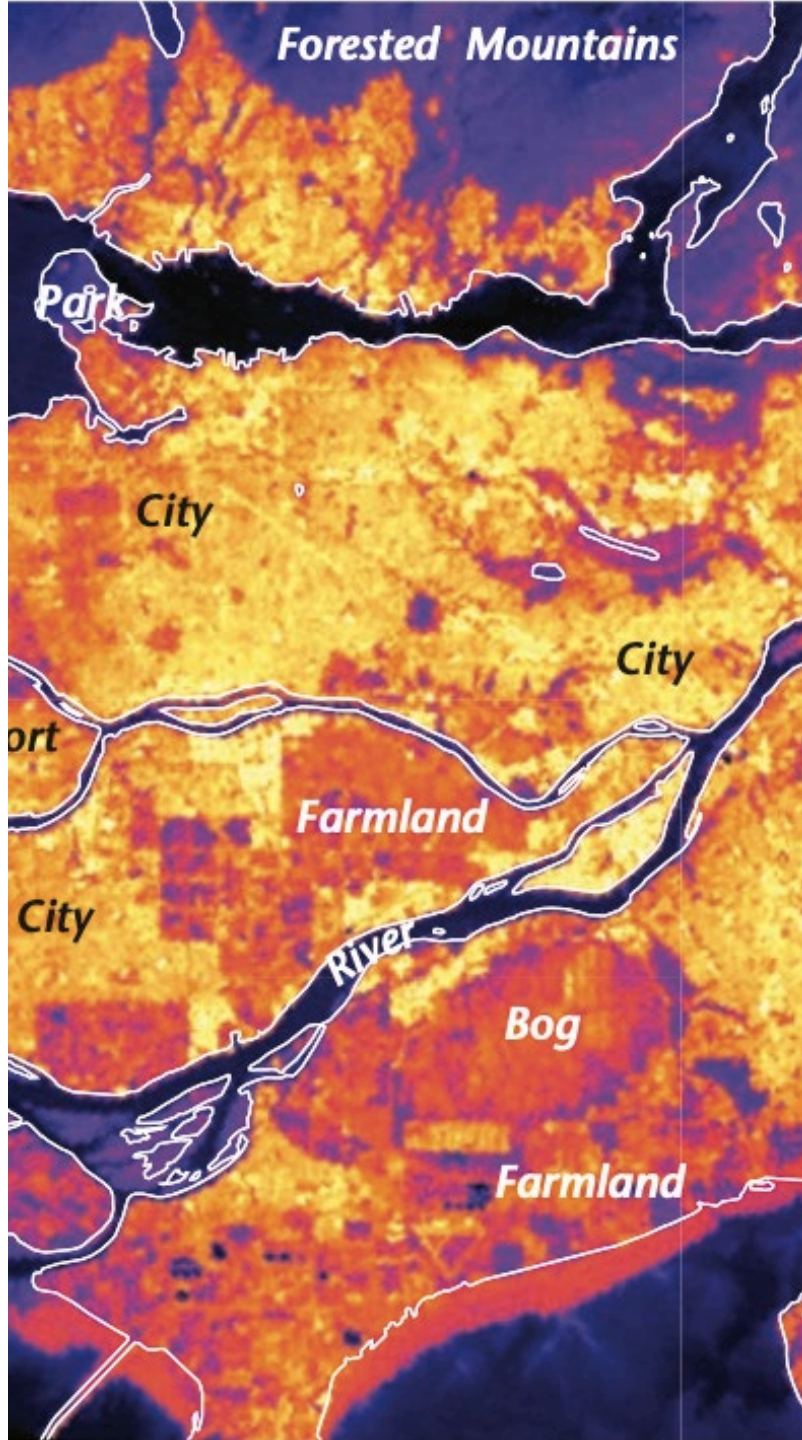
$$\tau_l = \frac{\rho_w \cdot D_0^2}{8 \cdot \rho \cdot \alpha_w \cdot (q_s - q_e)} \quad (8-11)$$

α_w, ρ_w should be at the film temperature $T_f = (T_e + T_s)/2$
 D_0 (m) – initial diameter of the droplet
 ρ (kg/m³) – density of water vapor mixture at T_f
 T_s (K) – wet-bulb temperature

- Initial temperature transience τ [K/s]: the time required for the droplet to warm up /cool down from its injection temperature (T_i) to final temperature (T_{wb}) can be estimated by calculating $\frac{dT}{dt}$:

$$\tau = -\frac{\rho_w \cdot c_v \cdot D}{6 \cdot (Q_H + Q_E)} \cdot (T_{wb} - T_s) \quad (8-12)$$





CONTENT:

- I. Introduction
- II. Urban forms of water bodies (lakes, ponds, rivers, fountains, sprays, roof ponds)
- III. Properties of water
- IV. Energy balance of the water bodies
 - Water reservoirs (ponds)
 - Water fountains
 - Water sprays
 - Example exercise

EPFL Water Sprays: Practice Exercise

Consider a water spray injecting water drops into the air of a courtyard, water droplets size are $50 \mu\text{m}$.

The injection temperature of the water drops is 20°C (293.15 K). The surrounding air temperature is at 35°C (308.15 K), relative humidity of 40%, and atmospheric pressure is 101.3 kPa (1 atm).



Using properties of air and water, determine for the following:

- Latent heat flux in $[\text{W}/\text{m}^2]$ due to the water spray evaporation
- Air temperature drop due to the evaporative cooling of the water spray in the courtyard volume of 10 m^3 , if water is supplied at the rate of $16 \text{ ml}/\text{s}$

The surface temperature of the water droplet and film temperature needs to be determined:

- $T_s \sim T_{wb}$ as surface temperature of droplets quickly reaches T_{wb} after a very short initial transient
- Wet-bulb temperature at 35°C and 40% RH is **24.5°C** (297.65 K)
- Film (average) temperature is $T_f = (T_{wb} + T_a)/2 = \frac{297.65+308.15}{2} = 302.9 \text{ K}$ (29.75 °C)

1. Molecular diffusivity of water vapor α_w (m^2/s)

at $p_a = 1 \text{ atm}$, $T_f = 302.9 \text{ K}$, $p_0 = 1 \text{ atm}$, $T_0 = 256 \text{ K}$

$$\alpha_w = 1.97 \times 10^{-5} \cdot \left(\frac{p_0}{p_a}\right) \cdot \left(\frac{T}{T_0}\right)^{1.685} = 1.97 \times 10^{-5} \cdot \left(\frac{1}{1}\right) \cdot \left(\frac{302.9}{256}\right)^{1.685} = \mathbf{0.2616 \times 10^{-4} \text{ m}^2/s}$$

2. Specific humidity q ($\frac{\text{kg of water vapor}}{\text{kg of moist air}}$):

$$q = 0.622 \frac{p_v}{p_a - 0.378 \cdot p_v} \quad (2-8b)$$

Partial pressure of water vapour pressure p_v is defined by knowing water-vapor saturation pressure $p_{v,sat}$ and relative humidity φ

$$p_v = \varphi \cdot p_{v,sat} \quad (2-10a)$$

$$p_{v,sat} = 611 \cdot e^{\frac{17.08 \cdot t}{234.18+t}} \quad (2-6a)$$

- State (e), surrounding air: $p_{v,e} = \varphi \cdot p_{v,sat} = \varphi_e \cdot 611 \cdot e^{\frac{17.08 \cdot t}{234.18+t}} = 0.4 \cdot 611 \cdot e^{\frac{17.08 \cdot 35}{234.18+35}} = 2252.1 \text{ Pa}$

$$q_e = 0.622 \frac{p_{v,e}}{p_a - 0.378 \cdot p_{v,e}} = 0.622 \frac{2252.1}{101300 - 0.378 \cdot 2252.1} = \mathbf{0.0139 \frac{kg}{kg}}$$

- State (s), on the surface of the drop: $p_{v,s} = \varphi \cdot p_{v,sat} = \varphi_s \cdot 611 \cdot e^{\frac{17.08 \cdot t}{234.18+t}} = 1 \cdot 611 \cdot e^{\frac{17.08 \cdot 24.5}{234.18+24.5}} = 3080.3 \text{ Pa}$

$$q_s = 0.622 \frac{p_{v,s}}{p_a - 0.378 \cdot p_{v,s}} = 0.622 \frac{3080.3}{101300 - 0.378 \cdot 3080.3} = \mathbf{0.0191 \frac{kg}{kg}}$$

EPFL Water Sprays: Practice Exercise Solution

3. Density of water vapor mixture at $T_f = 302.9$ K:

Expressed analogously to solids per Eqn. (7-4):

$$\rho = \rho_w \cdot \phi_w + \rho_a \cdot \phi_a$$

- Density of air at $T_f = 302.9$ K: $\rho_a = 1.165 \frac{kg}{m^3}$
- Density of water at $T_f = 302.9$ K: $\rho_w = 996 \frac{kg}{m^3}$

Volume fraction of water vapor is expressed analogously to Eqn. (7-2):

$$\phi_v = \frac{m_{moist\ air}}{V_{moist\ air}} \times \frac{V_{water}}{m_{water}} \quad \leftarrow \begin{array}{l} \text{can be} \\ \text{expressed as} \end{array} \quad \phi_v = \frac{v}{q \cdot \rho_w}$$

Humidity by volume per Eqn. (2-11) $v = \frac{(p_{v,e} + p_{v,s})/2}{462 \cdot (t_f + 273.15)} = \frac{(2252.1 + 2342.6)/2}{462 \cdot (302.9)} = 0.019 \frac{kg}{m^3}$

Specific humidity at the average temperature $T_f = 302.9$ K: $q = \frac{q_{v,e} + q_{v,s}}{2} = \frac{0.0139 + 0.0191}{2} = 0.01654 \frac{kg}{kg}$

Finally, $\phi_w = \frac{0.019}{0.01654 \cdot 996} = 0.0012 \longrightarrow \rho = \rho_w \cdot \phi_w + \rho_a \cdot (1 - \phi_w) = 996 \cdot 0.0012 + 1.165 \cdot (1 - 0.0012) = 2.36 \frac{kg}{m^3}$

EPFL Water Sprays: Practice Exercise Solution

▪ **Latent heat transfer:** $Q_E = L_v \cdot E$ where $E = \rho \cdot \alpha_w \cdot \frac{(q_s - q_e)}{h} = \rho \cdot \alpha_w \cdot \frac{(q_s - q_e)}{D/2}$

○ Latent heat of vaporization at film (average) temperature $T_f = 302.9 \text{ K}$: $L_v = 2430 \frac{\text{kJ}}{\text{kg}}$

○ Latent heat flux: $Q_{E,50} = 2430 \times 10^3 \cdot 2.36 \cdot (0.2616 \times 10^{-4}) \cdot \frac{2 \cdot (0.0191 - 0.0139)}{50 \times 10^{-6}} = 31'204 \frac{\text{W}}{\text{m}^2} \approx 31.2 \frac{\text{kW}}{\text{m}^2}$

▪ **Air temperature drop due to evaporative cooling:**

○ Number of droplets injected: $N = \frac{\dot{m}}{V_i}$ $N_{50} = \frac{16 \times 10^{-6}}{\frac{1}{6} \cdot \pi \cdot (50 \times 10^{-6})^3} \sim 244.586 \times 10^6 \text{ drops/s}$

○ Latent heat of N number of drops in [W]: $Q_{E,N} = Q_E \cdot N \cdot A_i = Q_E \cdot N \cdot (\pi D^2)$ $Q_{E,N,50} = 59.91 \text{ kW}$

○ Air temperature decrease is determined by the heat storage change of the air due to the energy taken for evaporation:

$$Q_{E,N} = \rho_a \cdot c_{p,a} \cdot V_a \cdot \Delta T \longrightarrow \Delta T = \frac{Q_{E,N}}{\rho_a \cdot c_{p,a} \cdot V_a} \quad \Delta T = \frac{59912.93}{1.146 \cdot 1007 \cdot 10} \approx 5.2 \text{ K}$$



**Thank you
for your attention**

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